

# HIGH-ENERGY CROSS SECTION FOR $e^+e^- \rightarrow W^+W^- \rightarrow 4 \text{ FERMIONS}(+\gamma)^\dagger$

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## Abstract

A recently suggested high-energy Born-form representation of the one-loop helicity amplitudes for  $e^+e^- \rightarrow W^+W^-$  is supplemented by including  $W^\pm$  decay and hard-photon radiation. Results for the differential and the total cross section for  $e^+e^- \rightarrow W^+W^- \rightarrow 4\text{fermions}(+\gamma)$  are given for the high-energy region of  $\sqrt{s} \gtrsim 500\text{GeV}$ .

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# 1 Introduction

In a recent paper on  $W$ -pair production in  $e^+e^-$  annihilation by two of us [1] a high-energy representation for the one-loop-corrected helicity amplitudes was given in simple analytic form. The one-loop helicity amplitudes in Ref. [1] are represented in the form of a Born approximation, the weak and the electromagnetic coupling constant of the Born approximation being replaced by (three) invariant amplitudes that depend on the kinematic variables of the reaction, the energy, momentum transfer and soft-photon cut-off. The representation of the helicity amplitudes differs from previous ones [2] with respect to the choice of the covariants and the corresponding invariant amplitudes. In distinction from Ref. [2], the explicit and fairly simple analytic form of the invariant amplitudes, written down in a few lines, is a novel feature of Ref. [1]. In the derivation of the expressions for the invariant amplitudes in Ref. [1], the high-energy expansions of the cross sections of Ref. [3] were extensively used.

In the present work we will supplement our recent results by including the decay of the produced  $W^\pm$  bosons as well as hard-photon radiation.

## 2 The ansatz for $e^+e^- \rightarrow W^+W^- \rightarrow 4\text{fermions}$ , generalities

For a theoretical description of  $W$ -pair production including  $W$  decay, as a natural starting point, we will use the (one-loop-corrected) amplitudes for on-mass-shell production and decay. The propagation of the decaying  $W$  bosons will be taken care of by Breit-Wigner denominators with constant widths. In so far as on-shell-production and -decay amplitudes are employed, while the invariant masses of the fermion pairs only enter via the Breit-Wigner denominators and the four-fermion phase space, such an ansatz corresponds to a narrow-width approximation for  $e^+e^- \rightarrow W^+W^- \rightarrow 4\text{fermions}$ . As on-shell production and decay amplitudes are gauge invariant, the so-defined amplitudes for  $e^+e^- \rightarrow W^+W^- \rightarrow 4\text{fermions}$  are gauge-independent by construction. The approach should be particularly reliable for invariant masses of the produced fermion pairs in the vicinity of the  $W^\pm$  mass,  $M_W$ .

Four-fermion production,  $e^+e^- \rightarrow 4\text{fermions}$ , in general, does not only proceed via the production and subsequent decay of a  $W$ -boson pair. Additional contributions are present that involve only a single  $W$  pole, or no pole at all (cf. e.g. [4]). With respect to  $W$ -pair production, such contributions form a non-resonant (more precisely, a non-doubly-resonant) background. It is expected that by applying suitable experimental cuts on the invariant masses of the produced fermion pairs, one may be able to eliminate such background contributions to a large extent when comparing theoretical predictions with experimental data.

From the point of view of four-fermion production the above ansatz, based on on-mass-shell amplitudes and  $W^\pm$  Breit-Wigner denominators, corresponds to a double-pole approximation (DPA) [5, 6, 7]. The gauge invariance of the full (one-loop) four-fermion-

production amplitude implies that the double-pole residue in this amplitude is gauge-parameter independent. In addition to the gauge-parameter-independent on-mass-shell-production and -decay amplitudes, the double-pole residue also contains so-called non-factorizable corrections [5], essentially due to soft-photon exchange connecting, e.g. a decay fermion from  $W^+$ -decay with the  $W^-$ -boson. Extensive investigations [8, 9] led to the result that such corrections vanish, when integrated over the invariant masses of the fermion pairs. Moreover, they are entirely negligible [8] at  $e^+e^-$  energies above a few hundred GeV, the region of energies considered in the present paper. Accordingly, non-factorizable corrections need not be considered any further.

A quantitative analysis of the error induced by neglecting non-doubly-resonant contributions to four-fermion production faces the difficulties of formulating a satisfactory gauge-invariant ansatz for  $e^+e^- \rightarrow 4\text{fermions}$  that includes all (doubly-resonant, singly-resonant and non-resonant) contributions. The simple replacement of  $[k_\pm^2 - M_W^2]^{-1}$  by  $[k_\pm^2 - M_W^2 + iM_W\Gamma_W]^{-1}$  in general violates gauge invariance as different parts in the gauge-invariant amplitude for  $e^+e^- \rightarrow 4\text{fermions}$  are differently affected by such a replacement [4]. Nevertheless, it seems that such a “fixed-width scheme” finds some justification in a “complex-mass scheme” [6]. For an estimate of the error induced by neglecting non-doubly-resonant contributions, a fixed width ansatz should be quite reliable. Accordingly, we will use it in section 3 to estimate the accuracy to be expected for the double-pole approximation of four-fermion production.

We note that how to go from the off-shell kinematics to the on-shell kinematics is not unique. In our analysis, in defining the double-pole residue, we fix the production solid angles of  $W^-$  and the decay solid angles of two of the final fermions (originating from  $W^+$  and  $W^-$ ) in the laboratory frame.

### 3 Estimating Background Contributions

With respect to W-pair production, four-fermion production not enhanced by two intermediate W resonances, as mentioned in Section 2, may be considered as a background. This background, in general, contains four-fermion production via only one intermediate W boson or via no W-resonance enhancements at all [4].

In this section, we estimate the importance of such non-doubly-resonant background contributions by comparing the results of tree-level calculations for  $e^+e^- \rightarrow W^+W^- \rightarrow 4\text{fermions}$  with the results for  $e^+e^- \rightarrow 4\text{fermions}$  based on the full set of diagrams contributing to the reaction; wherever W poles appear in the full set of diagrams, as mentioned, fixed widths are introduced in the denominators (“fixed width scheme”). Specifically, we will consider the  $u\bar{d}\bar{c}s$  final state, thus comparing  $e^+e^- \rightarrow W^+(\rightarrow u\bar{d})W^-(\rightarrow \bar{c}s)$  with  $e^+e^- \rightarrow u\bar{d}\bar{c}s$ . We will also briefly comment on the semileptonic decay,  $e^+e^- \rightarrow W^+(\rightarrow u\bar{d})W^-(\rightarrow e\bar{\nu}_e)$ .

Our numerical results are collected in Table 1. They are based on the input parameters  $M_W = 80.22\text{GeV}$ ,  $M_Z = 91.187\text{GeV}$  and  $\alpha = 128.07^{-1}$ , and were obtained by employing GRACE [10], the automatic computation system for electroweak processes. For simplicity,

| Line  | $\sqrt{s}$   | 500 GeV                | 1 TeV                  | 2 TeV                  |
|---|--|------------------------|------------------------|------------------------|
| 1   | $\sigma(e^+e^- \rightarrow W^+W^-)$  | 7.458                  | 2.785                  | $9.421 \times 10^{-1}$ |
| Zero width approximation  |  |                        |                        |                        |
| 2   | $\sigma \times BR(W^+ \rightarrow u\bar{d}) \times BR(W^- \rightarrow \bar{c}s)$ | $8.289 \times 10^{-1}$ | $3.094 \times 10^{-1}$ | $1.047 \times 10^{-1}$ |
| Breit-Wigner, full four-fermion phase space                                       |  |                        |                        |                        |
| 3   | $\sigma(e^+e^- \rightarrow W^+(\rightarrow u\bar{d})W^-(\rightarrow \bar{c}s))$  | $8.291 \times 10^{-1}$ | $3.097 \times 10^{-1}$ | $1.046 \times 10^{-1}$ |
| 4   | $\sigma(e^+e^- \rightarrow u\bar{d}\bar{c}s)$                                    | $8.466 \times 10^{-1}$ | $3.248 \times 10^{-1}$ | $1.124 \times 10^{-1}$ |
| 5   | Difference $\Delta$ in %   | 2.1 %                  | 4.9%                   | 7.5%                   |
| Breit-Wigner, restricted phase space, $ \sqrt{k_\pm^2} - M_W  \lesssim 5\Gamma_W$ |  |                        |                        |                        |
| 6   | $\sigma(e^+e^- \rightarrow W^+(\rightarrow u\bar{d})W^-(\rightarrow \bar{c}s))$  | $7.264 \times 10^{-1}$ | $2.713 \times 10^{-1}$ | $9.16 \times 10^{-2}$  |
| 7   | $\sigma(e^+e^- \rightarrow u\bar{d}\bar{c}s)$                                    | $7.275 \times 10^{-1}$ | $2.717 \times 10^{-1}$ | $9.19 \times 10^{-2}$  |
| 8   | Difference $\Delta$ in %   | 0.1 %                  | 0.1 %                  | 0.3 %                  |

Table 1: Tree-level results in [pb] for  $W^+W^-$ -mediated four-fermion production (specifically for the  $u\bar{d}\bar{c}s$  final state) compared with four-fermion production including (non-doubly-resonant) background for different phase-space cuts.

the fermion masses are neglected in the calculation. The explicit calculation by **GRACE** shows that the effect of the finite fermion masses is about 0.2%.

Line 1 of Table 1 gives the cross sections for (stable) W-pair production for various energies. The results in line 2 of Table 1 are obtained by multiplication of the W-pair-production cross sections by the appropriate branching ratios, BR. The branching ratios for  $W^+ \rightarrow u\bar{d}$  and  $W^- \rightarrow \bar{c}s$  are given as  $BR = 1/3$ . Line 2 thus corresponds to the zero-width approximation of  $\Gamma_W \rightarrow 0$ ,  $BR = \text{const}$ , i.e.,

$$\frac{M_W \Gamma_W}{|k_\pm^2 - M_W^2 + iM_W \Gamma_W|^2} BR \rightarrow \pi \delta(k_\pm^2 - M_W^2) \cdot BR. \quad (3.1)$$

The results of line 2 practically agree with the results of line 3 that are obtained by (sevenfold Monte Carlo) integration of the Breit-Wigner resonance shape over the four-fermion phase space using the on-shell W-pair-production amplitudes. The results in line 4 are based on the full set of four-fermion-production diagrams with fixed widths inserted where appropriate. The difference between the results in line 3 and the four-fermion production,  $e^+e^- \rightarrow u\bar{d}\bar{c}s$ , of line 4, depending on the energy, lies between 2% and 8%, as indicated in line 5. Clearly, the entire four-fermion production exceeds the production via the decay of both  $W^+$  and  $W^-$  by 2% to 8%.

Finally, we enhance the relative contribution of the  $W^+$  and  $W^-$  resonances by imposing the restriction

$$|\sqrt{k_\pm^2} - M_W| \lesssim 5\Gamma_W, \quad (3.2)$$

on the masses of the fermion pairs,  $\sqrt{k_\pm^2}$ , when integrating over the four-fermion phase space. The results in lines 6 to 8 show that background contributions are reduced to roughly 0.1% to 0.3%, if the cut (3.2) is imposed. Upon employing the cut (3.2), one

can forget about all the four-fermion states that do not originate from the decay of two  $W$ -bosons.

We add a comment at this point on the background and its reduction by the cut (3.2), if the  $W$  bosons are identified by a decay mode different from the one discussed above. In particular, we have looked at the leptonic mode,  $W^- \rightarrow e\bar{\nu}_e$ . In this decay mode, without cut, the difference  $\Delta$  between  $\sigma(e^+e^- \rightarrow W^+(u\bar{d})W^-(e\bar{\nu}_e))$  and  $\sigma(e^+e^- \rightarrow u\bar{d}e\bar{\nu}_e)$ , the quantity corresponding to line 5 of Table 1, becomes very large, 81.9%, 96.3% and 99.3%, for  $\sqrt{s} = 500$  GeV, 1 TeV and 2 TeV, respectively. When the cut (3.2) on  $k_\pm^2$  is applied, however, we can reduce the background almost completely and the difference  $\Delta$  corresponding to line 8 of Table 1 becomes, 0.9%, 0.4% and 0.4% respectively.

Accordingly, it seems appropriate to apply theoretical as well as experimental cuts of the kind (3.2), when comparing theoretical predictions with experiment. This procedure largely circumvents the gauge-invariance issues inherently connected with a general theoretical treatment of four-fermion production. The procedure is conceptually simple and fully sufficient, moreover, for all practical purposes, as it is mainly the empirical test of the  $W$ -boson properties that one is interested in.

## 4 The ansatz for $e^+e^- \rightarrow W^+W^- \rightarrow 4\text{fermions}$ , details.

In one-loop order the helicity amplitudes of the process  $e^+(\sigma_+)e^-(\sigma_-) \rightarrow W_{\lambda_+}^+(f_1(\tau_1)\bar{f}_2(\bar{\tau}_2))W_{\lambda_-}^-(f_4(\tau_4)\bar{f}_3(\bar{\tau}_3))$  are given by

$$\mathcal{H}(\sigma_+, \sigma_-; \tau_1, \bar{\tau}_2; \tau_4, \bar{\tau}_3) = \sum_{\lambda_+, \lambda_-} \frac{\mathcal{H}(e^+(\sigma_+)e^-(\sigma_-) \rightarrow W_{\lambda_+}^+(f_1(\tau_1)\bar{f}_2(\bar{\tau}_2))W_{\lambda_-}^-(f_4(\tau_4)\bar{f}_3(\bar{\tau}_3)))}{K_+ K_-}, \quad (4.1)$$

where  $\sigma_\pm$  is the positron(electron) helicity,  $\lambda_\pm$  is the helicity of  $W^\pm$ , and  $\tau_i(\bar{\tau}_i)$  are helicities of the final fermions (antifermions). The explicit form of the numerator of (4.1) is given by

$$\begin{aligned} & \mathcal{H}(e^+(\sigma_+)e^-(\sigma_-) \rightarrow W_{\lambda_+}^+(f_1(\tau_1)\bar{f}_2(\bar{\tau}_2))W_{\lambda_-}^-(f_4(\tau_4)\bar{f}_3(\bar{\tau}_3))) \\ &= +\mathcal{H}_{Born}^{e^+e^- \rightarrow W^+W^-}(\sigma_+, \sigma_-; \lambda_+, \lambda_-)\mathcal{H}_{Born}^{W^+ \rightarrow f_1\bar{f}_2}(\lambda_+; \tau_1, \bar{\tau}_2)\mathcal{H}_{Born}^{W^- \rightarrow f_3\bar{f}_4}(\lambda_-; \tau_4, \bar{\tau}_3) \\ &+ \delta\mathcal{H}^{e^+e^- \rightarrow W^+W^-}(\sigma_+, \sigma_-; \lambda_+, \lambda_-)\mathcal{H}_{Born}^{W^+ \rightarrow f_1\bar{f}_2}(\lambda_+; \tau_1, \bar{\tau}_2)\mathcal{H}_{Born}^{W^- \rightarrow f_3\bar{f}_4}(\lambda_-; \tau_4, \bar{\tau}_3) \\ &+ \mathcal{H}_{Born}^{e^+e^- \rightarrow W^+W^-}(\sigma_+, \sigma_-; \lambda_+, \lambda_-)\delta\mathcal{H}^{W^+ \rightarrow f_1\bar{f}_2}(\lambda_+; \tau_1, \bar{\tau}_2)\mathcal{H}_{Born}^{W^- \rightarrow f_3\bar{f}_4}(\lambda_-; \tau_4, \bar{\tau}_3) \\ &+ \mathcal{H}_{Born}^{e^+e^- \rightarrow W^+W^-}(\sigma_+, \sigma_-; \lambda_+, \lambda_-)\mathcal{H}_{Born}^{W^+ \rightarrow f_1\bar{f}_2}(\lambda_+; \tau_1, \bar{\tau}_2)\delta\mathcal{H}^{W^- \rightarrow f_3\bar{f}_4}(\lambda_-; \tau_4, \bar{\tau}_3), \end{aligned} \quad (4.2)$$

and

$$K_\pm \equiv k_\pm^2 - M_W^2 + iM_W\Gamma_W, \quad (4.3)$$

where  $\sqrt{k_\pm^2}$  is the invariant mass of the fermion pair. All the helicity amplitudes appearing in (4.2) are evaluated on the mass-shell of the  $W^\pm$ -bosons. The first line corresponds to

the Born amplitude, while the second to the fourth line represent the one-loop corrections to the production and the  $W^\pm$ -decay subprocesses. The spin correlation is fully taken into account in formulation (4.1).

In order to regularize the infrared singularity, we include the soft-photon contribution in the virtual-loop corrections.

In terms of the helicity amplitudes (4.2), the differential cross section is given by

$$d\sigma = \frac{1}{2s}(2\pi)^{-7} \frac{1}{512} \sum' \int d\cos\theta dk_+^2 dk_-^2 d\cos\hat{\theta}_2 d\hat{\varphi}_2 d\cos\hat{\theta}_4 d\hat{\varphi}_4 \frac{|\vec{k}_\pm| |\mathcal{H}(e^+(\sigma_+)e^-(\sigma_-) \rightarrow W_{\lambda_+}^+(f_1(\tau_1)\bar{f}_2(\bar{\tau}_2))W_{\lambda_-}^-(f_4(\tau_4)\bar{f}_3(\bar{\tau}_3)))|^2}{E_{beam} [(k_+^2 - M_W^2)^2 + \Gamma_W^2 M_W^2][(k_-^2 - M_W^2)^2 + \Gamma_W^2 M_W^2]}, \quad (4.4)$$

where  $\sum'$  indicates the spin sum over the final fermions as well as the spin average of the initial  $e^+e^-$  pair, and it is understood that only the corrections of up to the order  $g^2$  are retained in the squares of the helicity amplitudes. The cosine of the W-production angle is denoted by  $\cos\theta$ , and  $\hat{\theta}_2(\hat{\theta}_4)$  and  $\hat{\varphi}_2(\hat{\varphi}_4)$  are the decay angles of fermion 2 (4) in the rest frame of the  $W^+(W^-)$ -boson. For further details on the notation and the kinematics, we refer to Appendix A. The magnitude of the three-momentum of the  $W^\pm$  in the  $e^+e^-$  center-of-mass frame is denoted by  $|\vec{k}_\pm|$ , where

$$|\vec{k}_\pm|^2 = \frac{1}{4s}(s^2 + k_+^4 + k_-^4 - 2s(k_+^2 + k_-^2) - 2k_+^2 k_-^2). \quad (4.5)$$

According to the analysis and discussion of Section 3, the range of the integration over the fermion-pair masses squared,  $k_\pm^2$ , is to be restricted in order to sufficiently reduce non-double-resonance four-fermion production. While too small an integration range will strongly reduce event rates, too large a range will increase background. We find that the five- $\Gamma_W$  choice (3.2) constitutes a reasonable compromise.

The seven-fold non-trivial integral (4.4) can be split into two integrals, the two-fold integral over the Breit-Wigner denominators defined by

$$I(s, M_W, \Gamma_W) \equiv \int_D dk_+^2 dk_-^2 \frac{|\vec{k}_\pm|}{E_{beam}} \frac{1}{[(k_+^2 - M_W^2)^2 + \Gamma_W^2 M_W^2][(k_-^2 - M_W^2)^2 + \Gamma_W^2 M_W^2]}, \quad (4.6)$$

with the integration region  $D$  restricted by the cut (3.2), and the remaining five-fold integral. The cross section (4.4) then becomes

$$d\sigma = \frac{1}{2s}(2\pi)^{-7} \frac{I(s, M_W, \Gamma_W)}{512} \sum' \int d\cos\theta d\cos\hat{\theta}_2 d\hat{\varphi}_2 d\cos\hat{\theta}_4 d\hat{\varphi}_4 \left| \mathcal{H}_{Born}^{e^+e^- \rightarrow W^+W^-} \mathcal{H}_{Born}^{W^+ \rightarrow f_1\bar{f}_2} \mathcal{H}_{Born}^{W^- \rightarrow f_3\bar{f}_4} + \delta\mathcal{H}_{Born}^{e^+e^- \rightarrow W^+W^-} \mathcal{H}_{Born}^{W^+ \rightarrow f_1\bar{f}_2} \mathcal{H}_{Born}^{W^- \rightarrow f_3\bar{f}_4} + \mathcal{H}_{Born}^{e^+e^- \rightarrow W^+W^-} \delta\mathcal{H}_{Born}^{W^+ \rightarrow f_1\bar{f}_2} \mathcal{H}_{Born}^{W^- \rightarrow f_3\bar{f}_4} + \mathcal{H}_{Born}^{e^+e^- \rightarrow W^+W^-} \mathcal{H}_{Born}^{W^+ \rightarrow f_1\bar{f}_2} \delta\mathcal{H}_{Born}^{W^- \rightarrow f_3\bar{f}_4} \right|^2. \quad (4.7)$$

| $E_{beam}(\text{GeV})$ | Born( $\text{GeV}^{-4}$ ) | one-loop( $\text{GeV}^{-4}$ ) |
|------------------------|---------------------------|-------------------------------|
| 200                    | $3.2646 \times 10^{-4}$   | $2.941 \times 10^{-4}$        |
| 300                    | $3.4343 \times 10^{-4}$   | $3.094 \times 10^{-4}$        |
| 400                    | $3.4917 \times 10^{-4}$   | $3.146 \times 10^{-4}$        |
| 500                    | $3.5180 \times 10^{-4}$   | $3.169 \times 10^{-4}$        |
| 600                    | $3.5322 \times 10^{-4}$   | $3.182 \times 10^{-4}$        |
| 700                    | $3.5407 \times 10^{-4}$   | $3.190 \times 10^{-4}$        |
| 800                    | $3.5462 \times 10^{-4}$   | $3.195 \times 10^{-4}$        |
| 900                    | $3.5500 \times 10^{-4}$   | $3.198 \times 10^{-4}$        |
| 1000                   | $3.5527 \times 10^{-4}$   | $3.201 \times 10^{-4}$        |

Table 2. The weight factor  $I(s, M_W, \Gamma_W)$  for the cut on  $k_+^2$  and  $k_-^2$  defined by (3.2).

The integral  $I$  thus plays the role of a weight factor originating from the Breit-Wigner propagators. Note that the integral  $I$  depends on  $s$  through  $|\vec{k}_\pm|/E_{beam}$ .

The decay width of the  $W^\pm$ -boson appearing in (4.7) is computed by GRACE [10]. The resulting width

$$\Gamma_W = 2.046 \text{GeV}, \quad (4.8)$$

includes the full one-loop radiative corrections with soft and hard bremsstrahlung of photon and gluon. The masses of the particles appearing in the loop calculation are listed in Appendix A. The numerical values of the integral  $I$  for the cut (3.2) on  $k_\pm^2$  are given in Table 2. The width  $\Gamma_W$  entering the Born value in table 2 is given by  $\Gamma_W = 1.942 \text{ GeV}$ , while the one-loop result is based on (4.8).

For the one-loop helicity amplitudes of  $e^+e^- \rightarrow W^+W^-$  in (4.7), we will use the high-energy-Born-form approximation (HEBFA) of Ref.[1]. In the HEBFA, the helicity amplitudes take the form [1]

$$\mathcal{H}(\sigma, \lambda_+, \lambda_-) = S_I^{(\sigma)} \mathcal{M}_I(\sigma, \lambda_+, \lambda_-) \delta_{\sigma,-} + S_Q^{(\sigma)} \mathcal{M}_Q(\sigma, \lambda_+, \lambda_-), \quad (4.9)$$

where the invariant amplitudes  $S_I^{(-)}(s, t, \Delta E)$  and  $S_Q^{(\pm)}(s, t, \Delta E)$  contain the one-loop virtual corrections as well as the soft-photon radiation with soft-photon-energy cut  $\Delta E$ . It is worth noting that the analytical formulae for the one-loop invariant amplitudes in (4.9) are very simple and can be written down in a few lines [1]. For definiteness and completeness, we will also compare with the result obtained by using the full one-loop-corrected amplitudes. Their analytical expressions were calculated in Refs.[11, 12]. For our numerical evaluation we will use the results of an independent calculation by one of us [13]. The numerical results of Ref.[13] agree well with the ones of Ref.[11].

Concerning the decay amplitudes for  $W^+ \rightarrow f_1 \bar{f}_2$  and  $W^- \rightarrow f_4 \bar{f}_3$  we can safely neglect fermion masses with a discrepancy of less than 0.3% [14]. In this approximation, up to one-loop order, the decay amplitudes can be expressed as (see also (6.18) of Ref.[15]),

$$\begin{aligned} \mathcal{H}^{W^+ \rightarrow f_1 \bar{f}_2}(\lambda_+; \tau_1, \bar{\tau}_2) &= \frac{g}{2\sqrt{2}} G^{(+)}(p_{W^+}, p_1, p_2) \mathcal{M}^{(+)}(\lambda_+; \tau_1, \bar{\tau}_2), \\ \mathcal{H}^{W^- \rightarrow f_4 \bar{f}_3}(\lambda_-; \tau_4, \bar{\tau}_3) &= \frac{g}{2\sqrt{2}} G^{(-)}(p_{W^-}, p_4, p_3) \mathcal{M}^{(-)}(\lambda_-; \tau_4, \bar{\tau}_3), \end{aligned} \quad (4.10)$$

where  $\lambda_{\pm}$ ,  $\tau_1$ ,  $\bar{\tau}_2$ ,  $\bar{\tau}_3$  and  $\tau_4$  are the helicities of  $W^{\pm}$ , and twice of the helicities of  $f_1$ ,  $\bar{f}_2$ ,  $\bar{f}_3$  and  $f_4$ , respectively. The basic amplitudes  $\mathcal{M}^{(\pm)}$  are defined by

$$\begin{aligned}\mathcal{M}^{(+)}(\lambda_+; \tau_1, \bar{\tau}_2) &= \bar{u}(p_1, -)\gamma_{\mu}(1 - \gamma_5)v(p_2, +)\epsilon_+^{\mu}(p_{W^+}, \lambda_+)\delta_{\tau_1, -}\delta_{\bar{\tau}_2, +}, \\ \mathcal{M}^{(-)}(\lambda_-; \tau_4, \bar{\tau}_3) &= \bar{u}(p_4, -)\gamma_{\mu}(1 - \gamma_5)v(p_3, +)\epsilon_-^{\mu}(p_{W^-}, \lambda_-)\delta_{\tau_4, -}\delta_{\bar{\tau}_3, +},\end{aligned}\quad (4.11)$$

in the massless limit, while the invariant amplitude  $G^{(\pm)}$  contains all the dynamic information of the process. The invariant amplitude  $G^{(\pm)}$  is normalized to unity at Born level

$$G_{Born}^{(\pm)} = 1. \quad (4.12)$$

At one-loop level,  $G^{(\pm)}$  receives contributions from virtual diagrams, the counter-term lagrangian and from the soft-photon radiation. Separating the dominant fermion contribution, it is expressed as,

$$G^{(\pm)} = 1 + \Delta\alpha(M_W^2) - \frac{c_W^2}{2s_W^2}\Delta\rho + \frac{1}{2}\Delta_{LL} + G^{(\pm, rest)}, \quad (4.13)$$

where the first term is the Born contribution,

$$\Delta\alpha(s) = \frac{\alpha}{3\pi} \sum_f Q_f^2 \log \frac{s}{m_f^2}, \quad (4.14)$$

$$\Delta\rho = \frac{g^2}{16\pi^2} \frac{3m_t^2}{4M_W^2}, \quad (4.15)$$

$$\begin{aligned}\Delta_{LL} &= \frac{2e^2}{16\pi^2} [\{Q_1^2 \log \frac{M_W^2}{m_1^2} + Q_2^2 \log \frac{M_W^2}{m_2^2} - Q_1^2 - Q_2^2 - 1\} \log \frac{(2\Delta E)^2}{M_W^2} \\ &\quad + (\frac{3}{2} - \log \frac{4p_{10}^2}{M_W^2})Q_1^2 \log \frac{M_W^2}{m_1^2} + (\frac{3}{2} - \log \frac{4p_{20}^2}{M_W^2})Q_2^2 \log \frac{M_W^2}{m_2^2}],\end{aligned}\quad (4.16)$$

and  $G^{(\pm, rest)}$  is the remaining part. The expression for  $\Delta_{LL}$ , due to the soft-photon radiation, should be compared with (3.7) of Ref.[1] for  $e^+e^- \rightarrow W^+W^-$ . In the present case, we have to evaluate the invariant amplitudes  $G^{(\pm)}$  not in the rest frame of the  $W^{\pm}$ -boson, but in the laboratory frame ( the c.m. frame of the  $e^+e^-$ -pair), and, consequently, for  $\Delta E$  we have to use the same numerical value as the one used in the  $W$ -pair production subprocess.

The full expressions for the basic amplitude,  $\mathcal{M}^{(\pm)}$ , and the remaining part,  $G^{(\pm, rest)}$ , are given in Appendices A and B, respectively.

## 5 Hard-photon radiation.

In DPA, the hard photon radiation can be treated in a way that is analogous to the above treatment of  $e^+e^- \rightarrow W^+W^- \rightarrow 4$  fermions. Since interference is negligible, the cross

section in DPA can be described by [7],<sup>‡</sup>

$$\begin{aligned}
d\sigma &= \frac{1}{2s} \left| \frac{\mathcal{H}^{e^+e^- \rightarrow W^+W^- \gamma} \mathcal{H}^{W^+ \rightarrow f_1 \bar{f}_2} \mathcal{H}^{W^- \rightarrow f_4 \bar{f}_3}}{K_+ K_-} \right|^2 \frac{dk_+^2 dk_-^2}{2\pi} d\Gamma_{\text{pro}}^\gamma d\Gamma_{\text{dec}}^+ d\Gamma_{\text{dec}}^- \\
&+ \frac{1}{2s} \left| \frac{\mathcal{H}^{e^+e^- \rightarrow W^+W^-} \mathcal{H}^{W^+ \rightarrow f_1 \bar{f}_2 \gamma} \mathcal{H}^{W^- \rightarrow f_4 \bar{f}_3}}{K_{+\gamma} K_-} \right|^2 \frac{dk_+^2 dk_-^2}{2\pi} d\Gamma_{\text{pro}} d\Gamma_{\text{dec}}^{+\gamma} d\Gamma_{\text{dec}}^- \\
&+ \frac{1}{2s} \left| \frac{\mathcal{H}^{e^+e^- \rightarrow W^+W^-} \mathcal{H}^{W^+ \rightarrow f_1 \bar{f}_2} \mathcal{H}^{W^- \rightarrow f_4 \bar{f}_3 \gamma}}{K_+ K_{-\gamma}} \right|^2 \frac{dk_+^2 dk_-^2}{2\pi} d\Gamma_{\text{pro}} d\Gamma_{\text{dec}}^+ d\Gamma_{\text{dec}}^{-\gamma} \quad (5.1)
\end{aligned}$$

where

$$\begin{aligned}
d\Gamma_{\text{pro}} &= \frac{1}{(2\pi)^2} \delta^4(k_1 + k_2 - k_+ - k_-) \frac{d^3 k_+ d^3 k_-}{2k_{+0} 2k_{-0}}, \\
d\Gamma_{\text{dec}}^+ &= \frac{1}{(2\pi)^2} \delta^4(k_+ - p_1 - p_2) \frac{d^3 p_1 d^3 p_2}{2p_{10} 2p_{20}}, \\
d\Gamma_{\text{dec}}^- &= \frac{1}{(2\pi)^2} \delta^4(k_- - p_3 - p_4) \frac{d^3 p_3 d^3 p_4}{2p_{30} 2p_{40}}, \\
d\Gamma_{\text{pro}}^\gamma &= \frac{1}{(2\pi)^5} \delta^4(k_1 + k_2 - k_+ - k_- - k_\gamma) \frac{d^3 k_+ d^3 k_- d^3 k_\gamma}{2k_{+0} 2k_{-0} 2k_{\gamma 0}}, \\
d\Gamma_{\text{dec}}^{+\gamma} &= \frac{1}{(2\pi)^5} \delta^4(k_+ - p_1 - p_2 - k_\gamma) \frac{d^3 p_1 d^3 p_2 d^3 k_\gamma}{2p_{10} 2p_{20} 2k_{\gamma 0}}, \\
d\Gamma_{\text{dec}}^{-\gamma} &= \frac{1}{(2\pi)^5} \delta^4(k_- - p_3 - p_4 - k_\gamma) \frac{d^3 p_3 d^3 p_4 d^3 k_\gamma}{2p_{30} 2p_{40} 2k_{\gamma 0}}, \quad (5.2)
\end{aligned}$$

and

$$\begin{aligned}
K_+ &= (p_1 + p_2)^2 - M_W^2 + iM_W \Gamma_W, \\
K_- &= (p_3 + p_4)^2 - M_W^2 + iM_W \Gamma_W, \\
K_{+\gamma} &= (p_1 + p_2 + k_\gamma)^2 - M_W^2 + iM_W \Gamma_W, \\
K_{-\gamma} &= (p_3 + p_4 + k_\gamma)^2 - M_W^2 + iM_W \Gamma_W. \quad (5.3)
\end{aligned}$$

$$(5.4)$$

In (5.1) the sum over the helicities of the intermediate  $W^\pm$ -bosons is implicitly assumed. Therefore, the full spin-correlation is incorporated also in the hard-photon bremsstrahlung process.

## 6 Numerical Results

In this section, we present the results for

$$e^+ e^- \rightarrow W^+(u\bar{d}) W^-(\bar{c}s) [+ \gamma], \quad (6.1)$$

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<sup>‡</sup>Hard-photon radiation in  $e^+ e^- \rightarrow 4 \text{ fermions} + \gamma$  at tree level, not based on a DPA was recently treated in Refs.[6, 18]

at one-loop level. As mentioned, we employ one-loop on-shell  $W^\pm$ - production and -decay amplitudes together with (fixed width) Breit-Wigner denominators and the cut  $M_W - 5\Gamma_W \leq \sqrt{k_\pm^2} \leq M_W + 5\Gamma_W$  on the fermion-pair masses. The cut strongly enhances the  $W^+W^-$  signal relative to contributions from the non-doubly-resonant background that is not taken into account in the calculation. From the point of view of four-fermion production directly observed experimentally, our results for (6.1) correspond to a double-pole approximation (DPA).

We summarize the main steps of the calculation:

i) For the amplitudes of  $e^+e^- \rightarrow W^+W^-$  at one-loop level we use the high-energy Born-form approximation (HEBFA) previously constructed [1]. Even though the comparison of the HEBFA with the full one-loop result for  $e^+e^- \rightarrow W^+W^-$  showed excellent agreement [1], we will nevertheless also present results for the reaction (6.1) that are based on the full one-loop amplitudes. The comparison of the results based on the HEBFA with the results from the full one-loop amplitudes will allow us to quantitatively state the accuracy of the HEBFA even when the decay of the W-boson is included.<sup>§</sup>

ii) For the decay amplitudes,  $W^\pm \rightarrow f\bar{f}$ , we use the full one-loop expression summarized in Appendix B. The fermion mass, with the exception of mass-singular terms, is neglected in the calculation of virtual and soft-photon corrections.

iii) The amplitudes for hard-photon emission are generated by using the algebraic manipulation program **GRACE** [10], and the necessary numerical integrations are carried out by employing the Monte Carlo routine **BASES** [19]. We require the error due to the Monte Carlo integration to be less than about 0.1 %. The independence of the result under the variation of the soft-photon cut  $\Delta E$  is verified by varying  $\Delta E$  between 1 GeV and 10 GeV.

The calculations are based on the following input parameters [in GeV],

$$\begin{aligned} M_Z &= 91.187, & M_W &= 80.22, & M_H &= 200, \\ m_u &= 0.062, & m_c &= 1.5, & m_t &= 175, \\ m_d &= 0.083, & m_s &= 0.215, & m_b &= 4.5. \end{aligned} \tag{6.2}$$

First of all, in Table 3 we consider the  $\cos\theta$  angular distribution of the produced  $W$  pairs at a fixed energy that is chosen as  $\sqrt{s} = 2E_{\text{beam}} = 1\text{TeV}$ . For easy reference, column 1 in Table 3 shows the Born-approximation results for  $e^+e^- \rightarrow W^+W^-$ . The second column of Table 3 includes the decay of the W-bosons, obtained by integrating the Breit-Wigner decay distributions over the restricted region (3.2). This restriction of the invariant mass of the fermion pairs leads to a slightly smaller cross section in column 2 than calculated by multiplication of the cross section for  $e^+e^- \rightarrow W^+W^-$  from column 1 by the square of the branching ratio,  $BR(W \rightarrow q\bar{q}) = 1/3$ .

The main result on the angular distribution is contained in the two one-loop columns of Table 3. The results are obtained for  $E_{\text{beam}} = 500\text{GeV}$  and an infrared cut-off  $\Delta E/E =$

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<sup>§</sup>In Ref.[1], we ignored the W decay. As different helicities enter the cross section for  $e^+e^- \rightarrow W^+W^- \rightarrow 4$  fermions with different weights, it is not a priori guaranteed that the accuracies excluding and including W decay are identical.

| $\cos \theta$ | $e^+e^- \rightarrow W^+W^-$ | $e^+e^- \rightarrow W^+(ud)W^-(\bar{c}s)$ | $e^+e^- \rightarrow W^+(ud)W^-(\bar{c}s) + \gamma$ |                        |              |
|---------------|-----------------------------|---|--|------------------------|--------------|
|               | Born                        | Born                                      | one-loop   |                        |              |
|               |                             |   | HEBFA  | exact                  | $\Delta(\%)$ |
| 0.95          | $5.981 \times 10^0$         | $5.827 \times 10^{-1}$                    | $2.900 \times 10^{-1}$                             | $2.878 \times 10^{-1}$ | 0.76         |
| 0.9           | $2.785 \times 10^0$         | $2.713 \times 10^{-1}$                    | $1.211 \times 10^{-1}$                             | $1.208 \times 10^{-1}$ | 0.24         |
| 0.8           | $1.207 \times 10^0$         | $1,176 \times 10^{-1}$                    | $4.557 \times 10^{-2}$                             | $4.557 \times 10^{-2}$ | 0.00         |
| 0.7           | $7.003 \times 10^{-1}$      | $6,826 \times 10^{-2}$                    | $2.383 \times 10^{-2}$                             | $2.385 \times 10^{-2}$ | -0.05        |
| 0.6           | $4.597 \times 10^{-1}$      | $4.483 \times 10^{-2}$                    | $1.437 \times 10^{-2}$                             | $1.438 \times 10^{-2}$ | -0.02        |
| 0.5           | $3.246 \times 10^{-1}$      | $3.165 \times 10^{-2}$                    | $9.429 \times 10^{-3}$                             | $9.435 \times 10^{-3}$ | -0.06        |
| 0.4           | $2.414 \times 10^{-1}$      | $2.352 \times 10^{-2}$                    | $6.570 \times 10^{-3}$                             | $6.576 \times 10^{-3}$ | -0.10        |
| 0.3           | $1.869 \times 10^{-1}$      | $1.821 \times 10^{-2}$                    | $4.798 \times 10^{-3}$                             | $4.808 \times 10^{-3}$ | -0.20        |
| 0.2           | $1.497 \times 10^{-1}$      | $1.458 \times 10^{-2}$                    | $3.645 \times 10^{-3}$                             | $3.651 \times 10^{-3}$ | -0.17        |
| 0.1           | $1.234 \times 10^{-1}$      | $1.201 \times 10^{-2}$                    | $2.855 \times 10^{-3}$                             | $2.861 \times 10^{-3}$ | -0.22        |
| 0.0           | $1.041 \times 10^{-1}$      | $1.013 \times 10^{-2}$                    | $2.292 \times 10^{-3}$                             | $2.297 \times 10^{-3}$ | -0.23        |
| -0.1          | $8.941 \times 10^{-2}$      | $8.695 \times 10^{-3}$                    | $1.872 \times 10^{-3}$                             | $1.876 \times 10^{-3}$ | -0.22        |
| -0.2          | $7.766 \times 10^{-2}$      | $7.551 \times 10^{-3}$                    | $1.542 \times 10^{-3}$                             | $1.544 \times 10^{-3}$ | -0.16        |
| -0.3          | $6.773 \times 10^{-2}$      | $6.586 \times 10^{-3}$                    | $1.268 \times 10^{-3}$                             | $1.269 \times 10^{-3}$ | -0.05        |
| -0.4          | $5.883 \times 10^{-2}$      | $5.721 \times 10^{-3}$                    | $1.031 \times 10^{-3}$                             | $1.030 \times 10^{-3}$ | 0.06         |
| -0.5          | $5.036 \times 10^{-2}$      | $4.897 \times 10^{-3}$                    | $8.174 \times 10^{-4}$                             | $8.148 \times 10^{-4}$ | 0.31         |
| -0.6          | $4.188 \times 10^{-2}$      | $4.073 \times 10^{-3}$                    | $6.202 \times 10^{-4}$                             | $6.155 \times 10^{-4}$ | 0.77         |
| -0.7          | $3.305 \times 10^{-2}$      | $3.214 \times 10^{-3}$                    | $4.364 \times 10^{-4}$                             | $4.291 \times 10^{-4}$ | 1.70         |
| -0.8          | $2.360 \times 10^{-2}$      | $2.295 \times 10^{-3}$                    | $2.680 \times 10^{-4}$                             | $2.573 \times 10^{-4}$ | 4.14         |
| -0.9          | $1.333 \times 10^{-2}$      | $1.296 \times 10^{-3}$                    | $1.215 \times 10^{-4}$                             | $1.074 \times 10^{-4}$ | 13.19        |

Table 3. The angular distribution of W-pair production at the energy  $\sqrt{s} = 2E_{beam} = 1$  TeV in units of  $pb$ . The first column shows the Born cross section for  $e^+e^- \rightarrow W^+W^-$ . The second column shows the results of treating W production and decay in Born approximation and integrating the Breit-Wigner distribution over the restricted interval (3.2). The third and the fourth column are obtained by using the one-loop amplitudes for production and decay, again, integrating the Breit-Wigner distribution over the restricted interval (3.2). A soft-photon cut  $\Delta E/E = 0.01$  is used for the one-loop results. The HEBFA is used for the third column and the full one-loop amplitudes are used for the fourth column. The last column gives the results for the relative deviation,  $\Delta$ , from (6.3).

0.01, i.e. for  $\Delta E = 5$  GeV. As mentioned, the one-loop amplitudes for W-pair production are supplemented by one-loop decay amplitudes, and the Breit-Wigner distributions are integrated over the restricted invariant-mass interval of 5 times the W-boson width,  $\Gamma_W$ , according to (3.2). The percentage deviations

$$\Delta = \frac{\frac{d\sigma}{d\cos\theta}(\text{HEBFA}) - \frac{d\sigma}{d\cos\theta}(\text{exact})}{\frac{d\sigma}{d\cos\theta}(\text{exact})}, \quad (6.3)$$

according to the last column of Table 3, stay below 3 per mill in most of the angular interval. As expected from the behavior of the three invariant amplitudes entering the HEBFA [1], the deviations between the approximation and the full one-loop results become largest (order of several percent) for very forward and backward angles. Nevertheless, the computationally very fast and intuitively simple HEBFA, for most of the range of the production angle, yields an excellent approximation of the full one-loop results for the differential cross section.

We turn to the total cross section as a function of the  $e^+e^-$  energy, obtained by integration over the angular distribution of the W-boson pair. Tables 4 and 5, respectively, show the cross sections obtained by integration over the full angular range of  $0^\circ < \theta < 180^\circ$  and over the restricted range of  $10^\circ < \theta < 170^\circ$ . As the forward peaking of the cross section increases strongly with energy, the angular cut with increasing energy leads to an increasingly stronger reduction of the integrated cross section. This is seen when comparing the cross sections in Tables 4 and 5. As a result of this angular cut, the accuracy of the HEBFA is strongly increased. The difference,

$$\Delta = \frac{\sigma(\text{HEBFA}) - \sigma(\text{exact})}{\sigma(\text{exact})}, \quad (6.4)$$

also shown in Tables 4 and 5, by removing the very forward- and backward- production angles is diminished by an order of magnitude, from about 3% to about 0.3%. The increase of accuracy is expected. As noted in connection with the results in Table 3, in the very forward and backward directions, the invariant amplitudes entering the HEBFA are less accurate [1].

In the high-energy limit, applying the cut  $10^\circ < \theta < 170^\circ$ , the total cross section deviates from the Born result only by a few percent. On the other hand, the cross section is strongly dominated by hard-photon radiation. This is shown in table 6. At a beam energy of  $E_{\text{beam}} = 200$  GeV, a fraction of 30% of the cross section contains a hard photon of energy  $E_\gamma > 10$  GeV, and this fraction rises strongly with increasing beam energy. At an energy of  $E_{\text{beam}} = 1$  TeV, a photon-energy cut of  $E_\gamma > 10$  GeV removes almost 75% of the cross section.

We finally add a very brief remark on the accuracy to be aimed at in future experiments in order to obtain a meaningful test of the non-Abelian structure of the electroweak theory. The non-Abelian structure enters both at tree level as well as at one loop. In Ref.[1], the invariant amplitudes of the HEBFA in (4.9) were represented as a sum of two parts. The

| $E_{\text{beam}}$ | $e^+e^- \rightarrow W^+W^-$ | $e^+e^- \rightarrow W^+(u\bar{d})W^-(\bar{c}s)$ | $e^+e^- \rightarrow W^+(u\bar{d})W^-(\bar{c}s) + \gamma$ |        |              |
|-------------------|-----------------------------|---|--|--------|--------------|
|                   | Born                        | Born  | one-loop   |        |              |
|                   |                             |   | HEBFA  | exact  | $\Delta(\%)$ |
| 200               | 8.698                       | 0.8467  | 0.8718   | 0.8794 | -0.9         |
| 300               | 5.091                       | 0.4958  | 0.5275   | 0.5262 | 0.2          |
| 400               | 3.384                       | 0.3295  | 0.3575   | 0.3533 | 1.2          |
| 500               | 2.433                       | 0.2370  | 0.2602   | 0.2556 | 1.8          |
| 600               | 1.844                       | 0.1795  | 0.1996   | 0.1951 | 2.3          |
| 700               | 1.452                       | 0.1413  | 0.1584   | 0.1543 | 2.7          |
| 800               | 1.177                       | 0.1145  | 0.1292   | 0.1259 | 2.6          |
| 900               | 0.9750                      | 0.09485   | 0.1080   | 0.1044 | 3.4          |
| 1000              | 0.8228                      | 0.08010   | 0.0915   | 0.0881 | 3.9          |

Table 4. The energy dependence of the  $(u\bar{d})(\bar{c}s)$ - production cross section in DPA. The second column is the Born cross section, while the third column gives the one-loop cross section including hard-photon radiation. The deviation,  $\Delta$ , according to (6.4), quantifies the discrepancy between the HEBFA and the full one-loop results.

| $E_{\text{beam}}$ | $e^+e^- \rightarrow W^+W^-$ | $e^+e^- \rightarrow W^+(u\bar{d})W^-(\bar{c}s)$ | $e^+e^- \rightarrow W^+(u\bar{d})W^-(\bar{c}s) + \gamma$ |         |              |
|-------------------|-----------------------------|---|--|---------|--------------|
|                   | Born                        | Born  | one-loop   |         |              |
|                   |                             |   | HEBFA  | exact   | $\Delta(\%)$ |
| 200               | 6.724                       | 0.6561  | 0.6746   | 0.6794  | -0.71        |
| 300               | 3.042                       | 0.2964  | 0.3109   | 0.3109  | 0.00         |
| 400               | 1.695                       | 0.1654  | 0.1725   | 0.1721  | 0.23         |
| 500               | 1.077                       | 0.1051  | 0.1085   | 0.1082  | 0.28         |
| 600               | 0.7440                      | 0.07262   | 0.07405  | 0.07383 | 0.30         |
| 700               | 0.5449                      | 0.05318   | 0.05349  | 0.05334 | 0.28         |
| 800               | 0.4162                      | 0.04063   | 0.04027  | 0.04015 | 0.30         |
| 900               | 0.3284                      | 0.03205   | 0.03136  | 0.03127 | 0.29         |
| 1000              | 0.2657                      | 0.02593   | 0.02505  | 0.02498 | 0.28         |

Table 5. As Table 4, but with a restriction on the  $W^+W^-$  production angle that is given by  $10^\circ < \theta < 170^\circ$ .

| $E_{\text{beam}}$       | 200    | 300    | 400    | 500    | 600     | 700     | 800     | 900     | 1000    |
|-------------------------|--------|--------|--------|--------|---------|---------|---------|---------|---------|
| $\sigma(E_\gamma < 10)$ | 0.4787 | 0.1928 | 0.0958 | 0.0544 | 0.03361 | 0.02195 | 0.01486 | 0.01033 | 0.00733 |
| $\sigma(E_\gamma > 10)$ | 0.2007 | 0.1181 | 0.0763 | 0.0538 | 0.04022 | 0.03139 | 0.02529 | 0.02094 | 0.01765 |
| $\sigma$                | 0.6794 | 0.3109 | 0.1721 | 0.1082 | 0.07383 | 0.05334 | 0.04015 | 0.03127 | 0.02498 |

Table 6. The total cross section for  $e^+e^- \rightarrow W^+(u\bar{d})W^-(\bar{c}s)\gamma$  is split into two parts according to the photon energy. The second line shows the total cross section with photon energy  $E_\gamma < 10$  GeV, while the third line shows the cross section with  $E_\gamma > 10$  GeV. The fourth line is the sum of these two, namely the total cross section. As for the results in Table 5, a cut in the  $W^+W^-$  production angle of  $10^\circ < \theta < 170^\circ$  is imposed.

first part<sup>¶</sup> contains only the leading fermion-loop corrections due to the light leptons and quarks and the heavy top quark, as well as the initial state radiation in leading-log approximation. The second additive part needs the full machinery of the non-Abelian electroweak theory. Taking into account only the aforementioned fermion loops, one finds that the cross section becomes larger than the full one-loop - corrected one. Quantitatively, at  $\sqrt{s} = 1$  TeV, with the fermion loops alone, the total cross-section becomes  $0.119 pb$ , a value almost 11% larger than the corresponding result of  $0.108 pb$  from table 5, while at  $\sqrt{s} = 2$  TeV, one finds  $0.030 pb$ , a value about 20% larger than the corresponding one of  $0.025 pb$ . An analogy to QED will be helpful to understand the negative sign of additional (primarily) bosonic loop corrections. In QED, photon exchange leads to an infrared divergence that is cancelled by (necessarily) positive soft-photon radiation. A similar cancellation mechanism involving (necessarily) positive  $Z_0$  emission, at ultra-high energies, will become relevant in the present case, thus allowing to understand the negative sign of the bosonic loop corrections. Their fairly large magnitude is the result of the log-squared terms appearing in these corrections [3, 1]. Accuracies of the order of magnitude of 10% to 20% will accordingly allow one to "see" the non-Abelian loops.

## 7 Conclusions

The production of four fermions in  $e^+e^-$  annihilation at high energies is dominated by the production and subsequent decay of two W-bosons. Our estimate at tree level shows that background contributions, wherein one of the fermion pairs, or both of them, do not originate from W decay, are of the order of 5% of the cross section in the high-energy region ( $400 GeV \lesssim \sqrt{s} \lesssim 2 TeV$ ) under consideration. Restricting the masses of the fermion pairs to the vicinity of the W-boson mass, however, removes the background apart from a negligible amount of less than 0.3% for the  $(u\bar{d})(\bar{c}s)$  channel and less than 0.9% for the  $(u\bar{d})(e\bar{\nu}_e)$  channel, the precise value depending on the energy being chosen. It is accordingly sufficient to concentrate on  $e^+e^- \rightarrow W^+W^- \rightarrow 4$  fermions and ignore background contributions in a refined calculation at one-loop level, even more so, as in four-fermion production the main interest lies in the test of the non-Abelian gauge-boson interaction of the electroweak theory. With respect to four-fermion production, evaluating  $e^+e^- \rightarrow W^+W^- \rightarrow 4$  fermions with appropriate cuts on the invariant masses of the fermion pairs amounts to a double-pole approximation (DPA).

We have presented results for  $e^+e^- \rightarrow W^+W^- \rightarrow (u\bar{d})(\bar{c}s) + \gamma$  at one-loop order in the high-energy limit. The results explicitly demonstrate that the HEBFA yields excellent results for the W-pair angular distribution, and for the total production cross section as well, if the very forward and backward regions are excluded. The results are very satisfactory for theoretical as well as practical reasons. Theoretically, the HEBFA is conceptually simple, as it includes all relevant virtual one-loop corrections within three

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<sup>¶</sup> Compare (3.3) and (3.4) as well as Figs.1 to 3 in Ref.[1].

invariant amplitudes that replace the weak and the electromagnetic coupling appearing at tree level. The HEBFA is of much practical importance, as the necessary computer time is strongly reduced with respect to a calculation that employs the full-one-loop results.

We finally gave a rough estimate of the accuracy future experiments are to aim at, in order to test the non-Abelian (loop) structure of the electroweak theory.

## 8 Acknowledgement

We would like to thank S. Dittmaier and J. Fujimoto for useful discussions.

## Appendix A. Notations and the basic amplitude for $W^\pm$ decay

In this Appendix, we show the explicit expression for the basic decay amplitudes  $\mathcal{M}^{(\pm)}$  of the  $W^\pm$ -boson defined by (4.11). Since we are in the laboratory frame (i.e. the  $e^+e^-$  center-of-mass frame) in which the produced  $W^\pm$  has a large energy, the decay amplitude explicitly depends on the four momentum of the  $W^\pm$ -boson through the soft photon energy cut. In accordance with the momentum assignment used in Refs.[1] and [16], we define the production angles and the corresponding  $W^\pm$  polarization vectors<sup>||</sup> as follows,

$$\begin{aligned} e^- : k_1^\mu &= E(1, 0, 0, 1), & e^+ : k_2^\mu &= E(1, 0, 0, -1), \\ W^- : k_-^\mu &= E_-(1, \beta_- \sin \theta, 0, \beta_- \cos \theta), & W^+ : k_+^\mu &= E_+(1, -\beta_+ \sin \theta, 0, -\beta_+ \cos \theta), \end{aligned} \quad (\text{A.1})$$

with

$$E_\pm = \frac{s + k_\pm^2 - k_\mp^2}{2\sqrt{s}}, \quad \beta_\pm = \sqrt{1 - \frac{k_\pm^2}{E_\pm^2}}. \quad (\text{A.2})$$

The decay angles are defined in the laboratory frame as

$$\begin{aligned} p_i^\mu &= E_i(1, \sin \theta_i \cos \phi_i, \sin \theta_i \sin \phi_i, -\cos \theta_i), & (i = 1, 2) \\ p_j^\mu &= E_j(1, \sin \theta_j \cos \phi_j, \sin \theta_j \sin \phi_j, \cos \theta_j), & (j = 3, 4) \end{aligned} \quad (\text{A.3})$$

and in the rest frame of the  $W^\pm$ -boson as

$$\begin{aligned} p_i^\mu &= \frac{\sqrt{k_+^2}}{2}(1, \sin \hat{\theta}_i \cos \hat{\varphi}_i, \sin \hat{\theta}_i \sin \hat{\varphi}_i, -\cos \hat{\theta}_i), & (i = 1, 2) \\ p_j^\mu &= \frac{\sqrt{k_-^2}}{2}(1, \sin \hat{\theta}_j \cos \hat{\varphi}_j, \sin \hat{\theta}_j \sin \hat{\varphi}_j, \cos \hat{\theta}_j). & (j = 3, 4) \end{aligned} \quad (\text{A.4})$$

In the latter frame, the phase space boundary becomes trivial,

$$-1 \leq \cos \hat{\theta}_i \leq +1, \quad 0 \leq \hat{\varphi}_i \leq 2\pi. \quad (\text{A.5})$$

After a simple algebra, we find that the energies of the decay products in the laboratory frame are related to those of the W-rest frame by

$$E_i = \frac{E_\pm}{2}(1 + \beta_\pm \cos \hat{\theta}_i), \quad (i = 1, 4) \quad (\text{A.6})$$

and the decay angles are related as follows

$$\begin{pmatrix} \sin \theta_i \cos \varphi_i \\ \sin \theta_i \sin \varphi_i \\ \mp \cos \theta_i \end{pmatrix} = \frac{1}{\beta_\pm + \cos \hat{\theta}_i} \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} \sin \hat{\theta}_i \cos \hat{\varphi}_i \\ \sin \hat{\theta}_i \sin \hat{\varphi}_i \\ \mp (\beta_\pm + \cos \hat{\theta}_i) \end{pmatrix}. \quad (\text{A.7})$$

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<sup>||</sup>This convention of polarization vectors is different from the one used by Ref.[3], in which  $\epsilon_-(-)$ ,  $\epsilon_+(0)$  and  $\epsilon_+(-)$  have opposite sign.

The polarization vector of the on-shell  $W^\pm$  with momenta specified by (A.1) are given by

$$\begin{aligned}\epsilon_-^\mu(0) &= \frac{2M_W}{\sqrt{s}}(\beta, \sin\theta, 0, \cos\theta), & \epsilon_+^\mu(0) &= \frac{2M_W}{\sqrt{s}}(-\beta, \sin\theta, 0, \cos\theta), \\ \epsilon_-^\mu(\pm) &= \mp \frac{1}{\sqrt{2}}(0, \cos\theta, \pm i, -\sin\theta), & \epsilon_+^\mu(\pm) &= \mp \frac{1}{\sqrt{2}}(0, -\cos\theta, \pm i, \sin\theta),\end{aligned}\quad (\text{A.8})$$

with

$$\beta = \sqrt{1 - \frac{4M_W^2}{s}}. \quad (\text{A.9})$$

The non-vanishing basic amplitudes for the on-shell  $W^\pm$  decay are given by

$$\begin{aligned}\mathcal{M}^{(-)}(\lambda; -, +) &= \sqrt{4E_3E_4\bar{\varphi}_-(p_4)}[\sigma_\mu\epsilon_-^\mu(\lambda)]\chi_+(p_3), \\ \mathcal{M}^{(+)}(\bar{\lambda}; -, +) &= \sqrt{4E_1E_2\bar{\varphi}_-(p_1)}[\sigma_\mu\epsilon_+^\mu(\bar{\lambda})]\chi_+(p_2),\end{aligned}\quad (\text{A.10})$$

where

$$\begin{aligned}\sigma_\mu\epsilon_-^\mu(\lambda) &= \begin{cases} \mp \frac{1}{\sqrt{2}} \begin{pmatrix} -\sin\theta & \cos\theta \pm 1 \\ \cos\theta \mp 1 & \sin\theta \end{pmatrix}, & \lambda = \pm \\ \gamma \begin{pmatrix} \beta + \cos\theta & \sin\theta \\ \sin\theta & \beta - \cos\theta \end{pmatrix}, & \lambda = 0 \end{cases} \\ \sigma_\mu\epsilon_+^\mu(\bar{\lambda}) &= \begin{cases} \mp \frac{1}{\sqrt{2}} \begin{pmatrix} \sin\theta & -\cos\theta \pm 1 \\ -\cos\theta \mp 1 & -\sin\theta \end{pmatrix}, & \bar{\lambda} = \pm \\ \gamma \begin{pmatrix} -\beta + \cos\theta & \sin\theta \\ \sin\theta & -\beta - \cos\theta \end{pmatrix}, & \bar{\lambda} = 0 \end{cases}.\end{aligned}\quad (\text{A.11})$$

and  $\varphi_-(p_1)$  and  $\chi_+(p_2)$  etc are the two component Weyl spinors

$$\begin{aligned}\varphi_-(p_1) &= \frac{1}{\sqrt{2(1 - \cos\theta_1)}} \begin{pmatrix} -\sin\theta_1 e^{-i\varphi_1} \\ 1 - \cos\theta_1 \end{pmatrix}, \\ \chi_+(p_2) &= \frac{1}{\sqrt{2(1 - \cos\theta_2)}} \begin{pmatrix} -\sin\theta_2 e^{-i\varphi_2} \\ 1 - \cos\theta_2 \end{pmatrix}, \\ \varphi_-(p_4) &= \frac{1}{\sqrt{2(1 + \cos\theta_4)}} \begin{pmatrix} -\sin\theta_4 e^{-i\varphi_4} \\ 1 + \cos\theta_4 \end{pmatrix}, \\ \chi_+(p_3) &= \frac{1}{\sqrt{2(1 + \cos\theta_3)}} \begin{pmatrix} -\sin\theta_3 e^{-i\varphi_3} \\ 1 + \cos\theta_3 \end{pmatrix}.\end{aligned}\quad (\text{A.12})$$

For the calculation of radiative corrections, the following values are used for our input parameters.

$$\begin{aligned}\alpha &= 1/137.036, \\ M_Z &= 91.187, & M_W &= 80.22, & M_H &= 200, \\ m_u &= 0.062, & m_c &= 1.5, & m_t &= 175, \\ m_d &= 0.083, & m_s &= 0.215, & m_b &= 4.5,\end{aligned}\quad (\text{A.13})$$

although, lepton masses and light quark masses are neglected except for the mass singular terms. The Born decay width of  $W^\pm$  becomes 1.942 GeV, while the decay width including order  $g^2$  and  $g_s^2$  corrections is 2.046 GeV.

## Appendix B. The invariant amplitude $G^{(\pm)}$ of $W^\pm$ decay

In this Appendix, we will present the expression of the invariant amplitude  $G^{(+)}$  for  $W^+ \rightarrow f_1 \bar{f}_2$ , where  $f_1$  and  $f_2$  have charge  $Q_1$  and  $Q_2 = Q_1 - 1$ , respectively. The invariant amplitude  $G^{(-)}$  for  $W^- \rightarrow f_4 \bar{f}_3$  is obtained by replacing the indices  $1 \rightarrow 3$  and  $2 \rightarrow 4$  in  $G^{(+)}$ . The invariant amplitude consists of the Born term, the contribution of the virtual corrections, the counterterm contribution and the soft-photon bremsstrahlung contribution. Decomposing it as,

$$G^{(+)}(p_W, p_1, p_2) = 1 + G^{(+,\text{virt})} + G^{(+,\text{ct})} + G^{(+,\text{soft})} \quad (\text{B.1})$$

we find after some calculation,

$$\begin{aligned} G^{(+,\text{virt})} = & +\frac{e^2}{16\pi^2} [Q_1^2 \log \frac{M_W^2}{m_1^2} + Q_2^2 \log \frac{M_W^2}{m_2^2}] \frac{\log \lambda^2}{M_W^2} \\ & +\frac{e^2}{16\pi^2} [\frac{Q_1^2}{2} \log^2(\frac{M_W^2}{m_1^2}) + \frac{Q_2^2}{2} \log^2(\frac{M_W^2}{m_2^2}) + 2Q_1^2 \log \frac{M_W^2}{m_1^2} + 2Q_2^2 \log \frac{M_W^2}{m_2^2}] \\ & +\frac{e^2}{16\pi^2} [\frac{4}{3} \pi^2 Q_1 Q_2 + 3 + \frac{\ell_1 \ell_2}{c_W^2 s_W^2} [2(2 + \frac{1}{c_W^2})(\log(c_W^2) - 1) \\ & \quad - 2(1 + \frac{1}{c_W^2})^2 \{Sp(-c_W^2) + \log(1 + c_W^2) \log(c_W^2)\}], \quad (\text{B.2}) \\ & +\frac{c_W^2}{s_W^2} [-2(c_W^2 + 2)J[1] + 5 + \frac{1}{c_W^2} + (2 + \frac{1}{c_W^2}) \log c_W^2 \\ & \quad + (2 + \frac{1}{c_W^2}) \int_0^1 dx \log(x^2 + \frac{1-x}{c_W^2})] ], \end{aligned}$$

where a tiny photon mass,  $\lambda$  is introduced in order to regularize the infrared singularity. The left-handed couplings  $\ell_1$  and  $\ell_2$  are given by

$$\ell_1 = \frac{1}{2} - s_W^2 Q_1, \quad \ell_2 = -\frac{1}{2} - s_W^2 Q_2, \quad (\text{B.3})$$

and  $J[1]$  is given by

$$J[1] = \int_0^1 \frac{dx}{1 - c_W^2 x} [-\log x + \log(x^2 + \frac{1-x}{c_W^2})]. \quad (\text{B.4})$$

$$G^{(+,\text{ct})} = -\frac{e^2}{16\pi^2} [Q_1^2 + Q_2^2 + 1] \frac{\log \lambda^2}{M_W^2} - \frac{e^2}{16\pi^2} [\frac{3}{2} Q_1^2 \log \frac{M_W^2}{m_1^2} + \frac{3}{2} Q_2^2 \log \frac{M_W^2}{m_2^2}]$$

$$\begin{aligned}
& +\frac{1}{2}\Delta\alpha(M_W^2) + \frac{e^2}{16\pi^2}[-\frac{1}{3} - 2Q_1^2 - 2Q_2^2 + \frac{\ell_1^2 + \ell_2^2}{4s_W^2 c_W^2} + \frac{1}{4s_W^2}] \\
& +\frac{1}{2}\delta Z_{Wf} - \frac{c_W^2}{2s_W^2}(\frac{\delta M_{Zf}^2}{M_Z^2} - \frac{\delta M_{Wf}^2}{M_W^2}),
\end{aligned} \tag{B.5}$$

where the explicit expression of  $\delta Z_{Wf}$  and  $\frac{\delta M_{Zf}^2}{M_Z^2} - \frac{\delta M_{Wf}^2}{M_W^2}$  can be found in Ref.[17].

$$\begin{aligned}
G^{(+, \text{soft})} = & -\frac{e^2}{16\pi^2} \left[ \{1 + Q_1^2 + Q_2^2 - Q_1^2 \log \frac{M_W^2}{m_1^2} - Q_2^2 \log \frac{M_W^2}{m_2^2}\} \log \frac{4(\Delta E)^2}{\lambda^2} \right. \\
& + \frac{Q_1^2}{2} \log^2 \left( \frac{4p_{10}^2}{m_1^2} \right) + \frac{Q_2^2}{2} \log^2 \left( \frac{4p_{20}^2}{m_2^2} \right) - Q_1^2 \log \frac{4p_{10}^2}{m_1^2} - Q_2^2 \log \frac{4p_{20}^2}{m_2^2} \\
& + (Q_1^2 + Q_2^2) \frac{\pi^2}{3} + 2(1 + Q_1 Q_2) Sp(1 - \frac{4E_1 E_2}{M_W^2}) \\
& - \frac{1}{\beta} \log \left( \frac{1 + \beta}{1 - \beta} \right) - 2 \log^2 \left( (1 + \beta) \frac{E}{M_W} \right) \\
& - 2Q_1 Sp(1 - \frac{2}{1 + \beta} \frac{E_2}{E}) - 2Q_1 Sp(1 - \frac{2}{1 - \beta} \frac{E_2}{E}) \\
& \left. + 2Q_2 Sp(1 - \frac{2}{1 + \beta} \frac{E_1}{E}) + 2Q_2 Sp(1 - \frac{2}{1 - \beta} \frac{E_1}{E}) \right],
\end{aligned} \tag{B.6}$$

where  $E_i$  and  $\beta$  are defined by (A.6) and (A.2) by setting  $k_\pm^2 = M_W^2$ . In (B.2), (B.5) and (B.6), the ultraviolet diverging parts which cancel from the sum are already discarded.

Removing the dominant fermionic contribution from the sum of (B.2), (B.5) and (B.6), one obtains  $G^{(\pm, \text{rest})}$ . It is again decomposed in three parts, which are given as follows.

$$G^{(+, \text{rest})} = \frac{e^2}{16\pi^2} (C^{(+, \text{virt})} + C^{(+, \text{ct})} + C^{(+, \text{soft})}), \tag{B.7}$$

where

$$\begin{aligned}
C^{(+, \text{virt})} &= +\frac{4}{3}\pi^2 Q_1 Q_2 + 3 \\
& - \frac{(1 - 2s_W^2 Q_1)(1 + 2s_W^2 Q_2)}{4c_W^2 s_W^2} [ + 2(2 + \frac{1}{c_W^2})(\log(-c_W^2) - 1) \\
& - 2(1 + \frac{1}{c_W^2})^2 \{ Sp(-c_W^2) + \log(1 + c_W^2) \log(c_W^2) \} ] \\
& + \frac{c_W^2}{s_W^2} [-2(c_W^2 + 2)J[1] + 5 + \frac{1}{c_W^2} + (2 + \frac{1}{c_W^2}) \log c_W^2 \\
& + (2 + \frac{1}{c_W^2}) \int_0^1 dx \log(x^2 + \frac{1}{c_W^2}(1 - x)) ] \\
& = \begin{cases} -2.1348 & \text{for leptonic decays} \\ -5.0730 & \text{for hadronic decays} \end{cases} \\
C^{(+, \text{ct})} &= -\frac{1}{3} - 2Q_1^2 - 2Q_2^2 + \frac{1 - 2s_W^2 + 2s_W^4(Q_1^2 + Q_2^2)}{8s_W^2 c_W^2} + \frac{1}{4s_W^2}
\end{aligned} \tag{B.8}$$

$$\begin{aligned}
& + \frac{16\pi^2}{e^2} \left[ \frac{1}{2} \delta Z_{Wf} - \frac{c_W^2}{2s_W^2} \left( \frac{\delta M_{Zf}^2}{M_Z^2} - \frac{\delta M_{Wf}^2}{M_W^2} \right) \right] + \log \frac{\lambda^2}{M_W^2} + \frac{3}{8s_W^4} \frac{m_t^2}{M_Z^2}, \quad (\text{B.9}) \\
C^{(+,soft)} = & -\frac{1}{2} Q_1^2 \log^2 \left( \frac{4E_1^2}{M_W^2} \right) + Q_1^2 \log \left( \frac{4E_1^2}{M_W^2} \right) - \frac{1}{2} Q_2^2 \log^2 \left( \frac{4E_2^2}{M_W^2} \right) + Q_2^2 \log \left( \frac{4E_2^2}{M_W^2} \right) \\
& - (Q_1^2 + Q_2^2) \frac{\pi^2}{3} - 2(1 + Q_1 Q_2) Sp \left( 1 - \frac{4E_1 E_2}{M_W^2} \right) \\
& + \frac{1}{\beta} \log \left( \frac{1+\beta}{1-\beta} \right) + 2 \log^2 \left( (1+\beta) \frac{E}{M_W} \right) \\
& + 2Q_1 Sp \left( 1 - \frac{2}{1+\beta} \frac{E_2}{E} \right) + 2Q_1 Sp \left( 1 - \frac{2}{1-\beta} \frac{E_2}{E} \right) \\
& - 2Q_2 Sp \left( 1 - \frac{2}{1+\beta} \frac{E_1}{E} \right) - 2Q_2 Sp \left( 1 - \frac{2}{1-\beta} \frac{E_1}{E} \right) ]. \quad (\text{B.10})
\end{aligned}$$

The last two terms in the second line of (B.9) remove the infrared singularity and the dominant top-quark mass effect present in the renormalization constants, making  $C^{(+,ct)}$  almost constant up to the Higgs-mass dependence.

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